Section 7.4 Dividing Radical Expressions

The general rule is: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{a}}$ when $\sqrt[n]{a}$ and $\sqrt[n]{a}$ are defined.

An example where the rule fails is: $\sqrt{\frac{-10}{-5}}$ because we are not allowed to have a negative in an "even root" (square root, fourth root, sixth root, etc)

$$\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$$

If we have variables involved, we should be sure they are defined for the "even root" problems. Generally, the text precedes a section of problems with "assume all variables are defined."

If this caveat is not included we have: $\sqrt{\frac{25}{x^2}} = \frac{5}{|x|}$ for example.

We normally do NOT allow final answers to have a radical on the bottom. $\sqrt{\frac{4}{5b}} = \frac{2}{\sqrt{5b}}$

To "cure" this problem we "rationalize" the fraction by multiplying the top and bottom by what ever it takes to remove the radical.

In the case of a square root, we need one more $\sqrt{5b}$ on the bottom to eliminate the radical so we multiply by the fraction $\frac{\sqrt{5b}}{\sqrt{5b}}$ (it has the value of one so it does not change the actual value of the fraction)

$$\sqrt{\frac{4}{5b}} = \frac{2}{\sqrt{5b}} \cdot \frac{\sqrt{5b}}{\sqrt{5b}}$$

so we get: $\frac{2\sqrt{5b}}{5b}$

If we had $\sqrt[3]{\frac{16x^3}{y^8}}$ simplifies to: $\frac{2x\sqrt[3]{2}}{y^2\sqrt[3]{y^2}}$

We need one more y inside the radical to give us 3 on the inside to eliminate the radical so we multiply by $\frac{\sqrt[3]{y}}{\sqrt[3]{y}}$ thus $\frac{2x\sqrt[3]{2}}{y^2\sqrt[3]{y^2}} \cdot \frac{\sqrt[3]{y}}{\sqrt[3]{y}}$ to give us $\frac{2x\sqrt[3]{2y}}{y^3}$.

Notice that the cube root of y has the same index as the cube root of 2 so both are contained in the same radical on the top.

Example:

$$\frac{\sqrt[3]{a}}{\sqrt[3]{9x}} \cdot \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}}$$ because we need one 3 and an x^2 to remove the radical

Our final answer is: $\frac{\sqrt[3]{3ax^2}}{3x}$

Section 7.5 Expressions containing several radical terms

When numbers or letters are involved in a radical, the radical represents a single number. In the same way an x squared represents a single number. We can add a few terms that all have x-squareds in them because they are like terms. Likewise, we can add (or subtract) several terms with the same radicals because we consider them like terms as well.

$6x^2 + 3x^2 - x^2$	$6\sqrt{7} + 3\sqrt{7} - \sqrt{7}$	$6\sqrt[3]{2} - 2x\sqrt[3]{2}$
$8x^2$	$8\sqrt{7}$	$(6 - 2x)\sqrt[3]{2}$

Sometimes the radicals do not appear to be the same.... $3\sqrt{8} - 5\sqrt{2}$

We know square root 8 is the same as $2\sqrt{2}$ so $3\sqrt{8}$ becomes $6\sqrt{2}$.

Our original problem is simplified as:

$$3\sqrt{8} - 5\sqrt{2}$$
$$6\sqrt{2} - 5\sqrt{2}$$
$$\sqrt{2}$$

We can multiply and divide radicals much like we do variables.

$$\sqrt{3}\left(x - \sqrt{5}\right)$$
$$x\sqrt{3} - \sqrt{15}$$

Notice that we have the x on the left of the radical. We were able to combine the 3 and the 5 into a 15 ONLY because both had the same index (ie. Both were square roots.)

When the indexes are different, we have a much more complicated job ahead

$$\sqrt{3}\left(x-\sqrt[3]{5}\right)$$
$$x\sqrt{3}-\sqrt{3}\cdot\sqrt[3]{5}$$
$$x\sqrt{3}-3^{\frac{1}{2}}\cdot 5^{\frac{1}{3}}$$
$$x\sqrt{3}-3^{\frac{3}{6}}\cdot 5^{\frac{2}{6}}$$
$$x\sqrt{3}-\left(3^3\right)^{\frac{1}{6}}\cdot\left(5^2\right)^{\frac{1}{6}}$$
$$x\sqrt{3}-(27)^{\frac{1}{6}}\cdot(25)^{\frac{1}{6}}$$
$$x\sqrt{3}-(27\cdot 25)^{\frac{1}{6}}$$
$$x\sqrt{3}-(675)^{\frac{1}{6}}$$
$$x\sqrt{3}-\sqrt[6]{675}$$

We needed to get our indexes the same. We used the common denominator of the fractional exponents to get a 6. Notice that the numerators became powers while we left the index on the outside.

When we had the same index, and ONLY when the index was the same, we were able to multiply the numbers inside the radical.

Example:

$\left(4\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-5\sqrt{2}\right)$ is the multiplication of two binomials. We use FOIL as you would expect.

$$4\sqrt{3}\cdot\sqrt{3}=4\cdot 3=12$$
$$4\sqrt{3}\cdot -5\sqrt{2}=-20\sqrt{6}$$
$$\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$$
$$\sqrt{2}\cdot -5\sqrt{2}=-10$$

The middle terms are like terms (as usual) and can be combined and the numbers 12 and – 10 can also be combined to give our final answer: $2-19\sqrt{6}$

We "rationalized" fractions before. We can do that with more difficult denominators but the method is the same. We multiply by whatever it takes to clear the radical on the bottom.

For these problems, we will use the "difference of squares" and the "sum and difference of cubes" to make this an easy job.

$$\frac{4}{\sqrt{3}+x}\cdot\frac{\sqrt{3}-x}{\sqrt{3}-x}$$
$$\frac{4\sqrt{3}-4x}{3-x^2}$$

$$\frac{4+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\cdot\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$$
$$\frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+2}{5-2}$$
$$\frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+2}{3}$$

$$\frac{4}{\sqrt[3]{3}-x}\cdot\frac{\left(\sqrt[3]{3}\right)^2+x\sqrt[3]{3}+x^2}{\left(\sqrt[3]{3}\right)^2+x\sqrt[3]{3}+x^2}$$
$$\frac{4\left(\sqrt[3]{3}\right)^2+4x\sqrt[3]{3}+4x^2}{\left(\sqrt[3]{3}\right)^3-x^3}$$
$$\frac{4\left(\sqrt[3]{3}\right)^2+4x\sqrt[3]{3}+4x^2}{3-x^3}$$

Example 6 page 461: $\frac{\sqrt[4]{(x+y)^3}}{\sqrt{x+y}}$ Notice the "base" $x + y$ is the same!

If the base had not been the same, we would still do the same thing but the final answer would not have collapsed as this one does.

First change to fractional exponents. Next, as before, change the fractional exponents so they have the same denominator, then simplify.

$$\frac{(x+y)^{\frac{3}{4}}}{(x+y)^{\frac{1}{2}}}=\frac{(x+y)^{\frac{3}{4}}}{(x+y)^{\frac{2}{4}}}=(x+y)^{\frac{3}{4}-\frac{2}{4}}=(x+y)^{\frac{1}{4}}=\sqrt[4]{x+y}$$

Notice that we put the expression back into a similar form as the original problem (ie. radicals)

An example where the bases are different:

$$\frac{\sqrt[4]{(x+y)^3}}{\sqrt{p+q}}=\frac{(x+y)^{\frac{3}{4}}}{(p+q)^{\frac{1}{2}}}=\frac{(x+y)^{\frac{3}{4}}}{(p+q)^{\frac{2}{4}}}=\frac{(x+y)^{\frac{3}{4}}}{(p+q)^{\frac{2}{4}}}\cdot\frac{(p+q)^{\frac{2}{4}}}{(p+q)^{\frac{2}{4}}}=\frac{\sqrt[4]{(x+y)^3(p+q)^2}}{p+q}$$

A better version of Example 9 page 462:

$$\frac{\left(a^2b^4\right)^{\frac{1}{3}}}{\left(ab\right)^{\frac{1}{2}}}$$

$$\frac{\left(a^2b^4\right)^{\frac{2}{6}}}{\left(ab\right)^{\frac{3}{6}}}$$

$$\frac{\left(a^4b^8\right)^{\frac{1}{6}}}{\left(a^3b^3\right)^{\frac{1}{6}}}$$

$$\left(\frac{a^4b^8}{a^3b^3}\right)^{\frac{1}{6}}$$

$$\left(ab^5\right)^{\frac{1}{6}}$$

$$\sqrt[6]{ab^5}$$